104 學年（下）博士班資格考（工程數學）

Problem 1. (20%)

(a) (10%) Find the eigenvalues and eigenfunctions of the boundary-value problem

$$x^2y'' + xy' + \lambda y = 0, \qquad y(1) = 0, \qquad y(5) = 0$$

(b) (10%) Find the square norm of each eigenfunction in problem (a).

Problem 2. (15%)

Use Laplace transform to solve the system of differential equation

$$x'' + y'' = e^{2t}$$
$$2x' + y'' = -e^{2t}$$
$$x(0) = 0, y(0) = 0$$
$$x'(0) = 0, y'(0) = 0$$

Problem 3. (15%)

Solve $\quad xy'' = y' + (y')^3$

Problem 4. (18%)

$$A = \begin{bmatrix} 4 & -1 & -2 \\ 2 & 1 & -2 \\ 1 & -1 & 1 \end{bmatrix}$$

Find the eigenvalues and their corresponding eigenvectors.

Problem 5. (16%)

Given $f(x, y, z) = 2x^2 - y^2 - z = 0$

(a) Find the unit normal vector at P(1,1,1)

(b) Find the tangential plane at P.

Problem 6. (16%)

Find the Fourier series of f on the given interval.

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 2, & 0 \le x < \pi \end{cases}$$